Adaptation of Geometric Functions for Calculating Tree Volume and Biomass in One Step

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Abstract

Stem volume and biomass are important variables frequently measured in forestry. Their direct measurements are impossible on standing trees and impracticable on felled logs. Consequently, stem volume and biomass are commonly estimated using selected established relationships between them and some directly measurable tree dimensions, such as diameter and height. One way of estimating them on standing trees and logs is by determination of their values, directly from detailed measurements of the trees or logs in question, using geometric formulae. The conventional way of using the geometric formulae involves at least two computational steps to calculate volume and at least four steps to calculate biomass. But, with some modifications, it is possible to calculate volume and biomass with these formulae in just one step of computation. The modifications involve building the necessary conversion factors; cross-sectional area formula and specific wood density into the various geometric formulae to enable them calculate the required stem content in a single step. The adapted formulae yield the same results as one would obtain by using the geometric formulae in the conventional way. By reducing the process to just one step, they save time, energy and space, reduce computational errors and avoid conversion from one unit of measurement to another and some approximations between steps that may affect the final result.

Key words: Huber's formula, Smalian's formula, Newton's formula, Conoid formula.

Introduction

Trees are the main focus of all forest inventories for timber and carbon; and tree stem content constitutes a very important quantity of interest in such forest assessments in view of its importance in forest management and utilization. Its quantification in cubic volume (m^3 or ft^3) or weight (kg) is always given serious attention because of the value of its products (Kauffman and Donato, 2012). Tree stem produces wood, which is the most prominent forest product in the world, given its numerous applications (Olajide and Eniang, 2000; Fuwape, 2005; Olajide and Udo, 2005) and its immense contributions to the national economies of many countries of the world. In fact, timber has been a primary source of capital for forested nations such as Canada, India, Indonesia, Italy, Malaysia, Norway, Sweden, the Russian Federation, Thailand and United States of America, among other countries, which have relied on their forests for sources of revenue, foreign exchange and equity for loans to get the necessary capital for building their industrial and agricultural capacities (Duerr, 1993; Durning, 1994). Even in modern economy, wood plays a part in more activities than does any other commodity, since there is hardly any industry that does not use wood or wood products somewhere in its manufacturing and marketing processes (Cunningham and Cunningham, 2004). Stem volume estimation is very useful for both research and practical purposes in forestry and contributes to the sustainable management of timber resources (Barrio-Anta, *et al*., 2007). Information on the quantity, quality and value of timber in the forest is always desired for timber management, sales and purchases. Generally, information on wood volume is very important for forest valuation, planning of sustained yield of timber harvest, monitoring and evaluation of management effects on forests, measuring the effect of experimental treatments on tree growth and developing growth and yield models for predicting sustained yield levels (Higman *et al.,* 2000; Panwar and Bhardwaj, 2012).

Monitoring and evaluation of forest biomass is becoming more important because of the vital roles played by forest ecosystems in regulating global carbon balance and mitigating global climate change (Tomppo *et al*., 2010). Biomass measurements have become crucial for determining the amount of carbon sequestered in vegetation and understanding the impacts of land-cover changes on carbon fluxes (Cole and Ewel, 2006; Heryati *et al*., 2011; Addo-Fordjour and Rahmad, 2013). This is because carbon mass in vegetation is a fractional part of biomass and can easily be calculated from it (Losi *et al*., 2003; Montagu *et al*., 2005; Kauffman and Donato 2012; Yuen *et al*., 2016). Generally, study of forest biomass is important in the study of ecosystem productivity, energy and nutrient flows, standing tree carbon and the effect of forest land dynamics on the global carbon cycle (Parresol, 1999; Yuen *et al*., 2016).

In estimating aboveground biomass and carbon stock of forest ecosystems, it is essential to measured trees thoroughly and accurately since they dominate the aboveground carbon pool and are an obvious indicator of land-use change and ecological condition (Santa and Tarazona, 2001; Kauffman and Donato, 2012). Additionally, trees have been identified as principal sinks of CO_2 among the forest species, (Sukhdev, 2010; ITTO, 2011). Estimation of tree stem biomass is very essential in quantifying total tree biomass. This is because the stem constitutes the highest proportion of total tree biomass (Ecometrica, 2011; Feliciano *et al*., 2014) and is more accessible and easier to measure than any other part of the tree. Sometimes the biomasses of other parts of the tree are estimated from stem biomass using known relationships between stem biomass and the biomasses of other parts of the tree (Ecometrica, 2011), especially when non-destructive method is used in estimating total tree biomass.

Stem volume and biomass are variables frequently measured in forestry, but are impossible to measure directly on standing trees. Even on fell trees, the direct measurement of these variables is impracticable due to the bulky nature of the stem, which requires large capacity and highly expensive instruments to handle. Aside from that, such exercise is very tedious and time consuming. Consequently, stem volume and biomass are commonly estimated using selected established relationships between them and other directly measurable tree dimensions, such as diameter and height. One way of estimating these

variables on standing trees and logs is by determination of their values directly from detailed field measurements of the trees or logs in question using geometric formulae.
The conventional method of direct estimation of stem content from diameter and height measurements using geometric formulae involves: (i) converting diameter measurements from centimetres to metres, since tree diameters are always measured in centimetres (cm) and volume in cubic metre (m^3), (ii) calculating cross sectional areas of the stem or log at the various points of diameter measurements, (iii) calculating volume using any of the geometric formulae and (iv) if biomass is required, converting volume to biomass by multiplying it by the wood density of the tree species. Wood density is usually expressed in gram per cubic centimetre (g/cm^3) and needs to be converted to kilogram per cubic metre (kg/m^3) before it can be used, since biomass is expressed in kilogram (kg). Thus, even if one bypasses the step for calculating cross sectional areas, it takes at least two computational steps to calculate volume and at least four steps to calculate biomass. But it is possible to calculate volume and biomass using these geometric formulae in just one step of computation, thus saving time and energy and avoiding conversion from one unit of measurement to another, computational errors and some approximations between steps that may affect the final result. The objective of this study was to build all the necessary conversion factors and the formula for calculating cross sectional area into the frequently used geometric formulae to facilitate their usage in calculating stem contents, even with the use of manual calculators.

Frequently used Geometric Formulae for Volume Estimation
The following are geometric formulae frequently used in forestry for calculating the volume of a stem or log:

Huber's formula: $V = \mathrm{h}(A_m)$ ……………………... (1)

Smalian's formula: $V = \frac{h}{2}(A_b + A_t)$ …………………………...................................... (2)

Newton's formula: $V = \frac{h}{6}(A_b + 4A_m + A_t)$ ………….. (3)

Conoid formula: $V = \frac{1}{3}A_b h$ .... (4)

Where; A_b = Cross sectional area at the base (m^2),
A_m = Cross sectional area at middle (m^2),
A_t = Cross sectional area at the top (m^2),
V = Cubic volume (m^3),
h = Height or length (m),
(Avery and Burkhart, 2002; Husch *et al*. 2003).
The Conoid formula is used for calculating the volume of the upper part of the tree stem, since upper logs usually approach the form of conoids (Clutter *et al*., 1983; Avery and Burkhart, 2002). Using the formulae as stated in equations (1) to (4) requires that the cross-sectional areas at the various points of measurements are calculated first, before the values are substituted into the geometric formulae. The cross-section of the stems of most trees approximates a circle, thus, its area is usually computed with the standard formula for calculating the area of a circle, stated as follows:

$A = \pi r^2 = \frac{\pi d^2}{4}$ …………………………………………………………………. (5)

Where; A = Cross-sectional area (m^2),
r = Radius (m),
d = Diameter (m) and
π = Constant
(Avery and Burkhart, 2002; Husch *et al*. 2003).
In forestry, since diameter instead of radius is the dimension of the stem usually measured, the derivative or modified version ($\frac{\pi d^2}{4}$) of the standard formula for calculating the area of a circle is normally adopted for calculating cross-sectional area of the stem.

Modification of the Formulae for Calculating Cross-Sectional Area and Volume
When cross-sectional area is only required for volume computation, the two steps for calculating cross-sectional area and volume can be accomplished in just one step if the formula for calculating cross-sectional area is built into the required geometric formulae for calculating volume as follows:

Huber's formula: $V = \mathrm{h}(A_m) = h\left(\frac{\pi d_m^2}{4}\right) = \frac{\pi d_m^2}{4}h$ ………... (6)

Smalian's formula: $V = \frac{h}{2}(A_b + A_t) = \frac{h}{2}\left(\frac{\pi d_b^2}{4} + \frac{\pi d_t^2}{4}\right) = \frac{\pi h}{8}\left(d_b^2 + d_t^2\right)$ .... (7)

Newton's formula $V = \frac{h}{6}(A_b + 4A_m + A_t) = \frac{h}{6}\left(\frac{\pi d_b^2}{4} + 4\frac{\pi d_m^2}{4} + \frac{\pi d_t^2}{4}\right) = \frac{\pi h}{24}(d_b^2 + 4d_m^2 + d_t^2)$ .... (8)

Conoid formula $V = \frac{1}{3}A_b h = \frac{1}{3} \times \frac{\pi d_t^2}{4} h = \frac{\pi d_t^2 h}{12}$ .... (9)

Where; d_b = Diameter at the base (m),
d_m = Diameter at the middle (m),
d_t = Top diameter measured at crown point (m),
V = Cubic volume (m^3)
h = Height or length (m)
π = Constant.

These modified versions of these geometric formulae have been used by some authors (Hamilton, 1988; Picard *et al*., 2012; Shamaki and Akindele, 2013). Using these geometric formulae as expressed in equations (6) to (9) for calculating volume, requires the conversion of diameter measurements from centimetre (cm) to metre (m) by dividing each diameter value by 100 before they are used in calculating volume to obtain the result in cubic metre if not, after the calculation the result must be divided by 10000 to have it in cubic metre. However, converting diameter measurements from centimetre to metre and calculating cross-sectional area can be accomplished in one step by building the conversion factor (100) into the formula for calculating cross-sectional areas as shown in equation (10).

$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times \frac{d}{100} \times \frac{d}{100} = \frac{\pi d^2}{40000}$ ……...……………….. (10)

Where; A = Cross-sectional area (m^2),
d = Diameter (cm) and
π = Constant

When the actual value of pi (π) is substituted into the equation, equation (10) can be simplified to:

$A = 0.00007854\mathrm{d}^2.$ ………………………………………………………………….. (11)

But the constant 0.00007854 is an approximation. The cross-sectional area formula with inbuilt conversion factor as expressed in equations (10) and (11) have been used by many authors for calculating cross-sectional areas (Avery and Burkhart, 2002; Husch *et al*. 2003; Laar and Akca, 2007; Etigale *et al*., 2014; Etigale *et al*., 2021).

When equation (10) is built into the various geometric formulae, they can be used to calculate volume directly from field measurements in just a single step and obtain results in cubic metres without converting from one unit to another as follows:

Huber's formula: $V = \mathrm{h}(A_m) = h\left(\frac{\pi d_m^2}{4}\right) = h\left(\frac{\pi d_m^2}{4 \times 100 \times 100}\right) = \frac{\pi d_m^2}{40000}\mathrm{h}$ ……..................... (12)

Smalian's formula $V = \frac{h}{2}(A_b + A_t) = \frac{h}{2}\left(\frac{\pi d_b^2}{40000} + \frac{\pi d_t^2}{40000}\right) = \frac{\pi h}{80000}\left(d_b^2 + d_t^2\right)$ .... (13)

Newton's formula $V = \frac{h}{6}(A_b + 4A_m + A_t) = \frac{h}{6}\left(\frac{\pi d_b^2}{40000} + 4\frac{\pi d_m^2}{40000} + \frac{\pi d_t^2}{40000}\right)$
$= \frac{\pi h}{240000}(d_b^2 + 4d_m^2 + d_t^2)$ .... (14)

Conoid formula $V = \frac{1}{3}A_t h = \frac{1}{3} \times \frac{\pi d_t^2}{40000} h = \frac{\pi d_t^2}{120000}\mathrm{h}$ .... (15)

Where; V = Cubic volume (m^3)
h = Height or length (m)
d_b = Diameter at the base (cm),
d_m = Diameter at the middle (cm),
d_t = Top diameter measured at crown point (cm).

Modification of the Formulae for Calculating Biomass

Biomass can also be calculated directly from field measurements in just a single step using these geometric formulae without going through the rigours of converting values from one unit to another and calculating cross-sectional area and volume. This can be accomplished by modifying the various geometric formulae and incorporating specific wood density term into the formulae as shown in equations (16) and (20). Note that specific wood density is expressed in g/cm^3 and the factor for converting it to kg/m^3 (1000) is also built into the formulae.

Huber's formula: $\boldsymbol{B = h(A_m) = h\left(\frac{\pi d_m^2}{40000}\right) \times 1000\rho = \frac{1000\pi h_1 \rho}{40000}\left(d_m^2\right) = \frac{\pi h \rho}{40}\left(d_m^2\right)}$ .... (16)

Smalian's formula: $\boldsymbol{B = \frac{h}{2}(A_b + A_t) = \frac{h}{2}\left(\frac{\pi d_b^2}{40000} + \frac{\pi d_t^2}{40000}\right) \times 1000\rho}$

$$= \frac{\mathbf{1000\pi h\rho}}{\mathbf{80000}}\left(\boldsymbol{d_b^2} + \boldsymbol{d_t^2}\right) = \frac{\boldsymbol{\pi h\rho}}{\mathbf{80}}\left(\boldsymbol{d_b^2} + \boldsymbol{d_t^2}\right) \quad \text{.. (17)}$$

Newton's formula $B = \frac{h}{6}(A_b + (4 \times A_m) + A_t) = \frac{h}{6}\left(\frac{\pi d_b^2}{40000} + \left(4 \times \frac{\pi d_m^2}{40000}\right) + \frac{\pi d_t^2}{40000}\right) \times 1000\rho$

$$= \frac{1000\pi h\rho}{240000}(d_b^2 + 4d_m^2 + d_t^2) = \frac{\pi h\rho}{240}(d_b^2 + 4d_m^2 + d_t^2) \quad \text{.................................. (18)}$$

Cnooid formula: $B = \frac{1}{3}A_t h = \frac{1}{3} \times \frac{\pi d_t^2}{40000} h \times 1000\rho = \frac{1000\pi h\rho}{120000} d_t^2 = \frac{\pi d_t^2 h\rho}{120}$ .... (19)

The total stem biomass of a standing tree can also be estimated in one simple step after the tree has been scaled from the base to the top. This can be done with the adapted Newton's (equation 18) and Conoid (equation 19) formulae. Equation (18) calculates biomass from the base of the tree to the Crown Point, while equation (19) takes care of the upper part of the tree, since upper logs usually approach the form of conoids. Adding the two equations together and simplifying the resulting equation as shown in equation (20), it produces a simple formula for calculating total main stem biomass of a standing tree (Etigale *et al*., 2021).

$$B = \frac{\pi h\rho(d_b^2 + 4d_m^2 + d_t^2)}{240} + \frac{\pi d_t^2 l\rho}{120} = \frac{\pi\rho(h(d_b^2 + 4d_m^2 + d_t^2) + 2ld_t^2)}{240}$$

$$= \frac{\pi\rho}{240}(h(d_b^2 + 4d_m^2 + d_t^2) + 2ld_t^2) \quad \text{…………………………… (20)}$$

Where; B = Biomass (kg)

h = Bole height or length (m)

d_b = Diameter at the base (cm),

d_m = Diameter at the middle (cm),

d_t = Top diameter measured at crown point (cm),

P = Wood density (g/cm^3),

l = Crown length (m).

The adapted formulae yield the same results, to the last decimal place, as one would obtain by using the geometric formulae in the conventional way (following all the computational steps and converting values from one unit to another) for calculating stem contents.

The adapted geometric formulae have some advantages over the long method of using the geometric formulae in the conventional way. The adapted geometric formulae provide the short cut for estimating stem contents directly from detailed measurements of stems of individual trees or sections of logs. The formulae do not require conversion of diameter measurements from cm to m and specific wood density values from g/cm^3 to kg/m^3 before they are used in estimating volume or biomass. The process of going through some computational steps before obtaining volume and biomass are reduced to just one step by adapting the geometric formulae to accomplish all conversions and computations at once. They are easy to use both with manual calculator and computer, as all the necessary values are keyed into the device at once. They save time, energy and space in using manual calculator or computer. Their use simplifies the use of geometric formulae for estimation of stem contents directly from detailed measurements of stems of standing trees or logs. No approximation is required before the final result, unlike the long procedure where at times some values with many digits need to be approximated before they are used for calculation in the next step.

Conclusion

The adapted formulae are derived from the normal geometric formulae for calculating volume. They are the same geometric formulae frequently used, the only difference is that the adapted formulae are enhanced by the various components, such as the necessary conversion factors, cross-sectional formula and specific wood density that have been built into them to enable them calculate the required stem content without going through the rigours involved in the conventional way of using the geometric formulae. The adapted formulae yield the same results as one would obtain by using the geometric formulae in the conventional way for calculating stem contents.

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